

$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x}$

$\frac{d}{dt} \left( \frac{\partial L}{\partial v^j} \right) = \frac{\partial L}{\partial x^j}$